One Day Workshop on Biological Membranes

October 2, 2009 in Regensburg

Organizers: Georg Dolzmann, Harald Garcke

Preliminary program

9:00 - 9:50	Matthias Röger *Minimal bending energy for confined structures*
9:50 - 10:20	Michael Helmers *Towards kinks in membranes with lateral phase separation*
10:20 - 11:00	Coffee break
11:00 - 11:50	Axel Voigt *Fluid-structure interaction in multicomponent biomembranes*
11:50 - 14:00	Lunch
14:00 - 14:50	Ricardo Nochetto *Fluid biomembranes: Modeling and computation*
14:50 - 15:20	Hassan Farshbaf-Shaker *Navier-Stokes models with applications to biological membranes*
15:20 - 16:00	Coffee break
16:00 - 16:50	Robert Nürnberg *Parametric finite element approximations for Willmore flow and Helfrich flow*

Abstracts

Hassan Farshbaf-Shaker (Universität Regensburg)

Thermodynamically consistent higher order phase field Navier-Stokes models with applications to biological membranes

In this talk we derive thermodynamically consistent higher order phase field models for the dynamics of vesicle membranes in incompressible viscous fluids. We start with basic conservation laws and an appropriate version of the second law of thermodynamics and obtain generalizations of models introduced by Du, Li and Liu and Jamet and Misbah. In particular we derive a stress tensor involving higher order derivatives of the phase field and generalize the classical Korteweg capillarity tensor.

Michael Helmers (Oxford)

Towards kinks in membranes with lateral phase separation

In the spontaneous curvature model for two-component lipid bilayers equilibrium shapes are described as surfaces minimising the bending energy of each component plus an interface contribution. Although this energy does not enforce smoothness across interfaces, kinks are usually not included in the analysis. In the talk we consider the above model for rotationally symmetric membranes and a one-dimensional analogue when kinks are not excluded a priori. We introduce a family of energies for smooth curves and phase fields, and we study its sharp interface limit.

Ricardo Nochetto (University of Maryland at College Park)

Fluid biomembranes: modeling and computation

We study two models for biomembranes. The first one is purely geometric since the equilibrium shapes are the minimizers of the Willmore energy under area and volume constraints. We present a novel method based on ideas from shape differential calculus. The second model incorporates the effect of the inside (bulk) viscous incompressible fluid and leads to more physical dynamics.

We use a parametric approach, which gives rise to fourth order highly nonlinear PDEs on surfaces and involves large domain deformations. We discretize these PDEs in space with an adaptive finite element method (AFEM), with either piecewise linear or quadratic polynomials, and a semi-implicit time stepping scheme. We employ the Taylor-Hood element for the Navier-Stokes equations together with iso-parametric elements, the latter being crucial for the correct approximation of curvature. We discuss several computational tools such as space-time adaptivity and mesh smoothing.

We also discuss a method to execute refinement, coarsening, and smoothing of meshes on manifolds with incomplete information about their geometry and yet preserve position and curvature accuracy. This is a new paradigm in adaptivity.

This work is joint with Andrea Bonito and M. Sebastian Pauletti.

Robert Nürnberg (Imperial College London)

Parametric finite element approximations for Willmore flow and Helfrich flow

Mathematical models of biomembranes are generally based on bending energies, such as the Helfrich energy, with the biomembranes being instances of equlilibrium shapes for these energies. In its simplest form, the Helfrich energy reduces to the Willmore functional. In addition, biological cell membranes are often characterized by different phases with different physical properties. Variational models take such effects into account by adding a line tension energy to the bending energy.

A practical approach to obtain equilibrium shapes for the considered energies is to consider their L^2-gradient flows. E.g. the Willmore flow arises as the L^2-gradient flow of the Willmore functional, while a volume and surface area preserving variant of this flow gives rise to the so-called Helfrich flow.

In this talk, we will discuss a recently introduced parametric finite element approximation of Willmore flow and Helfrich flow, that allows for the inclusion and for the control of the tangential motion of the parametric triangulation. This tangential motion results in well-behaved meshes in practice.

Finally, we will present some preliminary results for the geometric evolution of surfaces involving line energies.

Matthias Röger (Univeristät Bonn)

Minimal bending energy for confined structures

We consider the minimization of the Willmore or elastica energy in the class of boundarys of smooth domains that are included in a given container. We analyze how the minimal bending energy is behaves as the length or area of the structures grows. In two dimensions we propose a diffuse (phase field) approximation of the constrained variational problem and present some numerical results.

Axel Voigt (TU Dresden)

Fluid-structure interaction in multicomponent biomembranes

We model multicomponent vesicles using an extended Helfrich-model and study the interplay of lipid concentration and morphology under the influence of fluid flow. Besides phase-field approximations also boundary elements are used.