Torsion algebraic cycles and etale cobordism

We show that the classical cycle class map from algebraic cycles to l-adic etale cohomology factors through a quotient of l-adic etale cobordism for smooth projective varieties over an algebraically closed field of positive characteristic. This yields a strong topological obstruction for cohomology classes to be algebraic. Moreover, this shows on the one hand that the complex Godeaux-Serre varieties used by Atiyah and Hirzebruch to find counterexamples for the integral Hodge conjecture may be used as examples for the non-surjectivity of the l-adic integral cycle map in positive characteristic as well. On the other hand we see that Totaro’s method to find nontrivial elements in the Griffiths group also works in positive characteristic.