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Algebraic Geometry I

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Exercise sheet 1

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Exercise 1. Let A be a ring, let $A[X_1, \ldots, X_n]$ be the polynomial ring in n variables over A and let $a_1, \ldots, a_n \in A$. Show that there is a canonical isomorphism of rings

$$A[X_1, \ldots, X_n]/ < X_1 - a_1, \ldots, X_n - a_n > \xrightarrow{\sim} A.$$

(4 points)

Exercise 2. Let k be a field and let $k[X]$ be the polynomial ring in one variable over k. Show that there are infinitely many prime ideals $\mathfrak{p} \subseteq k[X]$. *(4 points)*

Exercise 3. Let $R \subseteq S \subseteq T$ be ring extensions. Show: If S is a finitely generated R-algebra and T is a finitely generated S-module, then T is a finitely generated R-algebra.
Is this conclusion also true under the weaker assumption that S is a finitely generated R-algebra and T is a finitely generated S-algebra?

(4 points)