University of Regensburg
Faculty of Sciences I – Mathematics

Prof. Dr. Uwe Jannsen
Patrick Forré

Algebraic Geometry I

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Exercise sheet 4

4th November 2008

In all the following let k be an algebraically closed field and $\mathbb{A}^n(k) = k^n$ the affine space endowed with the Zariski topology.

Exercise 1. Let $U = \mathbb{A}^2(k) \setminus \{(0,0)\}$ be the plane without the origin. Show that the restriction map

$$\mathcal{O}(\mathbb{A}^2(k)) \to \mathcal{O}(U)$$

is an isomorphism.

(4 points)

Exercise 2. Let $U = \mathbb{A}^2(k) \setminus \{(0,0)\}$ again be the plane without the origin. Show that the quasi affine variety U is not affine, i.e., is not isomorphic to an affine variety.

(4 points)

Exercise 3. Let A be an arbitrary ring and $f \in A$. Show: There is a canonical isomorphism of rings

$$A_f \cong A[T]/ < f \cdot T - 1 > .$$

(4 points)

Exercise 4. Let U be quasi affine variety. Show: The units of the ring $\mathcal{O}(U)$ of regular functions of U are the regular functions ϕ with $\phi(P) \neq 0$ for all $P \in U$.

(4 points)

http://www.uni-regensburg.de/Fakultaeten/nat_Fak_I/Jannsen/home/UebungWS0809/AlgGeoI.htm Submission: 11th November 2008