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Progressive Bayes: A New State Estimation Framework for Nonlinear Systems

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Often, the internal state sequence of a dynamic system is not directly available and has to be reconstructed from the system input sequence and a measurement sequence supplied by sensor devices. Here, we assume discrete–time systems with continuous states, where the measurements and the inputs are nonlinearly related to the states.

The goal is to make an estimate of the state sequence available at every time step. Of course, this estimate should incorporate the information contained in all the input and measurement samples collected up to that time step. Instead of storing all data and reprocessing them at every time step, a recursive estimator is preferred, that uses some kind of sufficient statistic as an *exact* compressed representation of the collected data. For that purpose, the probability density functions of the state are well suited. Once they are available, almost any type of point estimate, e.g. mean, mode, or median, can be derived.

In the case of continuous states, however, the exact probability density functions characterizing the state estimate are in general either not feasible or not well suited for recursive processing. Hence, approximations of the true densities are generally inevitable. Several different choices for representing the density of the state estimate are possible. A Gaussian mixture approximation is especially convenient as its moments can be calculated analytically.

In this talk, a new estimator is introduced, which systematically minimizes a measure of deviation between the true and the approximate density by adapting *both structure and parameters* of the approximation density. Hence, the estimation problem is converted to an optimization problem, which consists of finding the parameters of the approximate density.

However, this is a complicated optimization problem with many local minima. For solving this minimization problem, a new type of homotopy continuation is proposed. For that purpose, a parameterized true density is introduced, which starts from a tractable density and *continuously* approaches the exact density to be approximated. Based on this type of progressive processing, the original optimization problem is converted into a corresponding system of ordinary differential equations. The desired optimal density parameters are then calculated by solving the differential equations over a finite "time" interval. Structural adaptation of the approximation density is performed during the progression in order to modify the local approximation capabilities of the mixture approximation by changing the number of components.

For different distance measures like the squared integral deviation between the true and the approximate density or the Kullback–Leibler–distance, specific expressions for the coefficients of the system of differential equations have been derived. Upon defining a specific type of approximation density, these expressions can be further simplified. A Gaussian mixture approximation is especially convenient and yields closed–form expressions for the desired coefficients.

The method has been applied to various processing steps, i.e., transformation of random variables, prediction step (time update) for nonlinear dynamic systems, and for including information of noisy measurements on the basis of a nonlinear measurement equation. The advantage for employing the new approach to the measurement step becomes immediately clear: Instead of including the measurement information at once, it is progressively included by modifying the prior density representing the prior knowledge gathered so far.

The proposed new state estimation approach is computationally efficient and provides systematic approximations of the underlying true densities and keeps the desired measure of deviation within a pre–specified tolerance band. If the approximation densities have separable kernels and the system nonlinearities are also separable or represented by a separable conditional density, the required multidimensional integrals can be reduced to the product of one–dimensional integrals.

Applications of new approach include the following real–world problems:

- localization in cellular radio networks, e.g. WLAN, DECT,
- source localization,
- and intention recognition in human–robot–cooperation.