Blockseminar: Surgery

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1   Introduction

The aim of this seminar is to give a detailed treatment of the smooth surgery exact sequence for manifolds of dimension 5 and higher and then to review extensions of this sequence to the piecewise linear and topological categories and to give a range of applications.

The main reference for the seminar is Wolfgang Lück's lecture notes, *A basic introduction to surgery* [L] but we will use various other sources.

1.1   Prerequisites

The seminar will assume not go into the following important subjects:

1. The general theory of smooth manifolds, submanifolds, normal bundles and tubular neighbourhoods: see e.g. [M-S, Ch. 11 & Ch. 18].

2. Participants are assumed to be familiar with the statement of the h-cobordism theorem and hopefully it's proof: see [M2].

An excellent place to learn the essential ideas for surgery remains [M1], in particular the first four sections.

2   Program

Note that the program naturally splits into various groups of talks. The speakers from these groups should co-operate with one another both in learning the material and in decided how to organise its presentation.

A Talks 1-3: The aims of these talks is to out-line the proof of the s-coboridism theorem assuming the audience is familiar with the proof of the h-cobordism theorem.
Regensburg

B Talks 4-5: Here the key homotopy theoretic aspects of manifolds are identified: Poincaré duality and its surprising consequence: the existence of the Spivak normal fibration.
Freiburg

C Talks 6-8: Working below the middle dimension we see how surgery and bordism are intimately related and how to make normal maps highly connected.
Regensburg

D Talks 9-11: These talks present the heart of surgery: the surgery obstruction map arising out of the challenge of performing desired surgeries on middle-dimensional homotopy classes. Subtle topology and complex algebra and elegantly gathered into a powerful synthesis here.
Edinburgh

E Talks 12-14: Here we step back a little and see how to assemble all of the previous hard work into the succient and powerful surgery exact sequence.
Münster 12 & 13 and Prof. Ammann 14

F Talks 15-18: Now we give applications and show how to work with the surgery exact sequence. Starting with exotic spheres we see the spaces fundamental spaces O, PL, TOP and G and their quotients, compute their homotopy groups and sometimes even their homotopy type. As a result we can calculate manifolds homotopy equivalent to S^n, $\mathbb{C}P^n$ and T^n (in appropriate categories).
Prof. Goette 15, Bonn 16 & 18 and Poznan 17

∗ *Note that generic references are to [L].*

2.1 Titles of Talks

- Monday

  1. The s-cobordism theorem I: [L, 1.1-1.3]

  2. The s-cobordism theorem II: [L, 1.3 & 1.4]

  3. The s-cobordism theorem III: [L, Ch. 2]

  4. Poincaré complexes: [L, 3.1]

  ∗ Exercise session 1

- Tuesday

  5. Spherical fibrations and the normal Spivak fibration: [L, 3.2.2 & 3.2.3]

  6. Normal maps and the Pontrjagin-Thom isomorphism: [L, 3.1 & 3.3]

  7. Surgery below the middle dimension : [L, 3.4]

  8. Intersections and self-intersections: [L, 4.1]

  ∗ Exercise session 2

- Wednesday

  9. Kernels and forms: [L, 4.2]

10. Even dimensional surgery obstructions: [L, 4.3 & 4.4]

11. Odd dimensional surgery obstructions: [L, 4.5 & 4.6]

∗ Afternoon excursion (hike?)

∗ Exercise session 3

- Thursday

12. Manifolds with boundary and simple surgery obstructions: [L, 4.7]

13. The structure set and Wall realisation: [L, 5.1]

14. The smooth surgery exact sequence: [L, 5.2, 5.3 & 6.1]

15. Exotic spheres: [L, 6.1-6.5 & 6.7]

∗ Exercise session 4

- Friday

16. The surgery exact sequence for TOP and PL: [L, 5.4, 6.6]

17. PL manifolds homotopy equivalent to $\mathbb{C}P^n$: [W, 14C], [M-M, 8C]

18. Fake tori: [W, 15A]

∗ Exercise session 5: Final questions and discussion

References

[B]   W. Browder, Surgery on simply-connected manifolds, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 65. Springer-Verlag, New York-Heidelberg 1972.

[K-L] M. Kreck and W. Lück, *Topological rigidity for non-aspherical manifolds*, Pure Appl. Math. Q. **5** (2009), no. 3, Special Issue: In honor of Friedrich Hirzebruch. Part 2, 873–914.

[L]   W. Lück, A basic introduction to surgery theory, ICTP Lecture Notes Series 9, Band 1, of the school "High-dimensional manifold theory" in Trieste, May/June 2001, Abdus Salam International Centre for Theoretical Physics, Trieste 2002. `http://131.220.77.52/lueck/data/ictp.pdf`

[M-M] I. Madsen and J. Milgram, The classifying spaces for surgery and cobordism of manifolds, Ann. Math. Stud., Princeton University Press 1979.

[MA] Manifold Atlas Project, *Exotic spheres*, Unrefereed page: `http://www.map.him.uni-bonn.de/Exotic_spheres`

[M1] J. Milnor, A procedure for killing homotopy groups of differentiable manifolds, Proc. Sympos. Pure Math., Vol. III (1961) 39–55.

[M2] J. Milnor Lectures on the h-cobordism theorem. Notes by L. Siebenmann and J. Sondow. Princeton University Press, Princeton, N.J. 1965.

[M-S] J. Milnor and Stasheff, Characteristic classes, Ann. of Math. Studies, No. 76. Princeton University Press, Princeton, N. J., University of Tokyo Press, Tokyo, 1974.

[R] A. A. Ranicki, Algebraic and geometric surgery, Oxford Mathematical Monographs. Oxford: Oxford University Press, 2002.

[W] C. T. C. Wall, Surgery on compact manifolds, Second edition. Edited and with a foreword by A. A. Ranicki. Mathematical Surveys and Monographs, **69**. American Mathematical Society, Providence, RI, 1999.