

# Seminar: Recent mathematical progress in General Relativity, Part II

Summer term 2014

Prof. Bernd Ammann

Dates: 10.4., 24.4., 8.5.,15.5.,22.5.,Tue 3.6.,~~5.6.~~,12.6.,26.6.,10.7.

**10.4. Talk no. 1,** *Current status and new aspects of the Penrose inequality*, NADINE GROSSE.  
[BrC], [Mar1]=ArXiv: 0906.5566, [CM]=ArXiv: 0911.0883.

**24.4. Talk no. 2,** *Mass of asymptotically hyperbolic manifolds*, MATTIAS DAHL.  
Definition of the mass invariants of asymptotically hyperbolic manifolds (and more general situations), discussion of positive mass theorems and the Penrose inequality in the asymptotically hyperbolic setting.

**Tue, 6.5. Talk no. 3,** *Cosmological censorship, a video*, We watch and discuss a presentation of Dafermos at the MSRI about the cosmic censorship conjecture.

**15.5. Talk no. 4,** *A generalization of Hawking's black hole topology theorem to higher dimensions*, BERND AMMANN.

The topology of horizons in arbitrary dimensions (in 3+1 dimensions they are spheres  $S^2$ , in general dimensions a manifold admitting positive scalar curvature. Explain the results and maybe the main ideas of proof of [GS]=ArXiv: gr-qc/0509107. Explain the results of [Ga1]=ArXiv: gr-qc/0608118.

**22.5.+ Tue 3.6. Supplementary Talk no. 1,** *Krein-Rutman theorem and applications to MOTS*, BERND AMMANN.

Explain the Krein-Rutman theorem and its proof, following [Smo, 11.C]. In this reference, the Krein-Rutman theorem is proved in a version which is sufficient for the Black hole topology theorem by Galloway. Alternative approaches with stronger statements, can be found in [NN], and [Dei, Chap. 6,§19, Theorem 19.2].

**12.6. and 26.6. Talk no. 5,** *Universal positive mass theorem*, NICOLAS GINOUX.  
[Her]=ArXiv: 1401.6009

**10.7. Talk no. 6,** *Universal spinor jet bundle*, NIKOLAI NOWACZYK.

**23.7 Talk no. 7,** *Conformal extensions for solutions to Einstein's equations*, OLAF MÜLLER.

## Seminar-Homepage

[http://www.mathematik.uni-r.de/ammann/lehre/2014s\\_\\_ART2](http://www.mathematik.uni-r.de/ammann/lehre/2014s__ART2)

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