

# Spectral geometry

Dates: 8.4, 15.4, 29.4, 6.5, 13.5, 20.5, 27.5, 3.6, 17.6, 24.6, 1.7, 8.7

Main reference will be [7] (Note also the many references therein).

1. Preliminaries from functional analysis and spectral theory [7, Chap 1.1, A.1.1]
2. Overview over some results on essential self-adjointness [7, Chap. 1.2-1.3]
3. Preliminaries on bounded geometry [7, App. 1]  
*(Should contain (at least) main definitions, Lemma 2.1, Theorems about decay of Green functions: Thms 2.2 and 3.7)*
4. Minimal and maximal operators on manifolds of bounded geometry [7, Chap. 1.4]  
*(Focuses on  $L^p$ -independence of spectra.)*
5. Preliminaries on Rigged Spaces and generalized eigenvalues [7, Appendix 2], [2, Chap. I.4], [1, Supplement 1.2.1-1.2.6] *(Basic idea in short: If one enlarges the domain of an operator to another Hilbert space  $\mathcal{H}^+$  in a reasonable way, then there will be a complete orthogonal system of 'generalized eigenvectors' of  $A$ .)*
6. Generalized eigenvalues - First example for a linear ODE on  $\mathbb{R}$  [1, Supplement 1.2.7 and references therein] and Generalized eigenvalues and manifolds of bounded geometry [7, Sec. 2.1]
7. Schnol-types Theorems [7, Sec. 2.2] *(When does a generalized eigenvalue actually belong to the  $L^2$ -spectrum? - Weak Bloch property)*
8. Discreteness of the spectrum of the Laplacian/Schrödinger operators on noncompact Riemannian manifolds [4], [1, Chap. 3.3.1]

From here on: The talks are independent and one can choose two out of it.

1. Limiting absorption principle [8], [5, Introduction] (And maybe more specialized literature by Kumura, and other ones in the references)
2. Hearing the Platycsoms [6]  
*(There is, up to scale, a unique isospectral pair of compact platycosms (=closed flat Riemannian 3-manifolds)).*
3. Isospectral towers of Riemannian manifolds  
*(Uses Sunada's method (algebraic) to isospectral manifolds. Probably one needs knowledge in arithmetic groups, local fields, and related structures)*

## References

- [1] Berezin, Shubin, *The Schrödinger equation*, Kluwer Academic Publishers Group
- [2] Gelfand, Wilenkin, *Verallgemeinerte Funktionen IV*, Hochschulbücher für Mathematik, Band 50, VEB Deutscher Verlag der Wissenschaften, Berlin, 1964
- [3] Linowitz, *Isospectral towers of Riemannian manifolds*, New York J. Math. 18 (2012) 451 – 461
- [4] Cianchi, Mazya, *On the discreteness of the spectrum of the Laplacian on noncompact Riemannian manifolds* J. Differential Geom. 87 (2011), no. 3, 469–491
- [5] Rodnianski, Tao, *Effective limiting absorption principles, and applications*, <http://arxiv.org/abs/1105.0873>
- [6] Rossetti, Conway, *Hearing the Platycsoms*, <http://arxiv.org/abs/math/0311470>
- [7] Shubin, *Spectral theory of elliptic operators on noncompact manifolds*, Méthodes semi-classiques, Vol. 1 (Nantes, 1991), Astérisque (207) 35–108, 1992
- [8] Tao, Blog entry on Limiting absorption principle <http://terrytao.wordpress.com/2011/04/21/the-limiting-absorption-principle/>