

# Seminar: Positive scalar curvature

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## Homotopy of $\text{Riem}^+(M)$

Let  $M$  be a compact manifold of dimension  $m$ , and let  $\text{Riem}^+(M)$  be the space of metrics with positive scalar curvature. The goal is to understand the homotopy groups of  $\text{Riem}^+(M)$ .

**Talk no. 1: Invariance under surgeries and bordisms. Bernd Ammann.**

We show that if  $N$  is obtained from  $M$  by surgery of dimension  $2, 3, \dots, m-3$ , then the surgery induces a homotopy equivalence from  $\text{Riem}^+(M)$  to  $\text{Riem}^+(N)$ , see [2, 5, 4]. This implies that the homotopy type of  $\text{Riem}^+(M)$  is in a certain sense a bordism invariant.

Additional comment: Some arguments on how to simplify bordisms are often hard to find in the literature in a correct way. A very helpful, recent reference for certain issues is [3, Prop. 2.1.1 and Cor. 2.1.2]

**Talk no. 2: Recent progress about  $\pi_k(\text{Riem}^+(M))$ .**

The goal is to explain [1]

## Seminar-Homepage

<http://www.mathematik.uni-regensburg.de/ammann/psc>

## Literatur

- [1] Boris Botvinnik, Johannes Ebert, and Oscar Randal-Williams. Infinite loop spaces and positive scalar curvature. ArXiv <http://arxiv.org/abs/1411.7408>.
- [2] Vladislav Chernysh. On the homotopy type of the space  $\mathcal{R}^+(M)$ . ArXiv <http://arxiv.org/abs/math/0405235>.
- [3] Michael Joachim Florian Hebestreit. Twisted spin cobordism. ArXiv <http://arxiv.org/abs/1311.3164>.
- [4] Mark Walsh. Metrics of positive scalar curvature and generalised Morse functions, Part I. *Mem. Amer. Math. Soc.*, 209, 2011. ArXiv version: <http://arxiv.org/abs/0811.1245>.
- [5] Mark Walsh. Cobordism invariance of the homotopy type of the space of positive scalar curvature metrics. *Proc. Amer. Math. Soc.*, 141:2475–2484, 2013. ArXiv version: <http://arxiv.org/abs/1109.6878>.