Seminar: Calabi Yau Conjecture and Special holonomy

Winter term 2015/16

Prof. Bernd Ammann

Date, Time, and Place

Monday 14-18, SFB BIO 1.1.34, irregular dates
For the dates and the speakers see the

Internal page of the seminar.

Overview

The main goal of the seminar is to understand the construction by Joyce of compact irrdeucible manifolds with holonomy G_2. We will follow the book [6]. The book is written in an efficient and self-contained way, so we want to follow the book to a large extent, mainly for the first part. Before we can understand this construction, we should go through the proof of Calabi conjecture by Yau. People only interested in Calabi-Yau manifolds, but not in special holonomy in general might follow the seminar up to this point. Once the construction of compact manifolds with holonomy G_2 is properly understood we could also discuss the construction of compact irreducible manifolds with holonomy Spin(7).
Note that Joyce has written two similar books: [6] and [7]. The content of the first part of the seminar is very close to the intersection of the two books. So you can find corresponding chapters in [7].
In the last part of the seminar we want to discuss current progress... (this part of the program will be written later).

1 Riemannian Holonomy

Talk no. 1: Definitions and basic theorems.
This talk will strongly depend on the prerequisites of the participants. It can be either ommited, one, two talks or even more. The audience should finally understand the essence of [6, Chapter 1–3].

2 Calabi-Yau manifolds

Talk no. 2: Kähler manifolds.
[6, Chapter 5]

Talk no. 3: Calabi conjecture.
[6, Chapter 6]

Talk no. 4: Calabi-Yau manifolds.
Explain [6, Chapter 7]. In particular: Crepant resolutions, orbifold Calabi-Yau manifolds, and related things. Less important are Sections 7.6 and 7.7. Section 7.6 provides many example of Calabi-Yau manifolds. Section 7.7. is not required to understand the Joyce construction, but needed later in the seminar. We could either do it at this point or do it later.

3 ALE and QALE spaces

Talk no. 5: ALE spaces. We introduce Riemannian metrics of ALE type. These are manifolds which are asymptotic to quotients of Euclidean space, see [4]=`http://arxiv.org/abs/math/9905041`.

Supplementary Talk no. 6: QALE spaces For many constructions of compact G_2 manifolds ALE manifolds are sufficient. For other examples and for manifolds with holonomy Spin(7) QALE manifolds are required. These are a bit more technical than ALE spaces, but if we understand them, we can construct more examples later. The speaker should take the analytical facts on QALE spaces as black boxes. See [7, Chap. 9] and also [5]=`http://arxiv.org/abs/math/9905043`.

4 Hyper-Kähler and quaternionic Kähler manifolds

Supplementary Talk no. 7: Hyper-Kähler manifolds and quaternionic Kähler manifolds
[6, Chapter 10]
Large parts of this section are not required for the following talks, but depending on the participants, it might be interesting to study them. There is one exception: We need Eguchi-Hanson spaces and some understanding of K3-surfaces. If we skip the talk above, someone should explain [6, Sections 10.2 and 10.3].

5 Constructions of compact examples with holonomy G_2

Supplementary Talk no. 8: General Facts about the exceptional Lie group G_2
This talk makes sense if and only if there are participants who did not follow the Lie group seminar in the summer term 2015. The goal would be to sumarize the essential properties of G_2. The ideal speaker would be a participant of the former Lie group seminar.

Talk no. 9: Manifolds with holonomy G_2.
General facts about G_2-structures and G_2-manifolds. [6, Sections 11.1 and 11.2],

maybe enriched by additional literature as e.g. Bryant's article `http://arxiv.org/abs/math/0305124`. The deformation theory in Bryant's article (Section 6) is not needed for the construction of G_2 metrics, but to understand the spcae of G_2 metrics (later talks). We treat deformation theory in this talk or later.

Talk no. 10: Construction of compact manifodls with holonomy G_2. The subject of this talk is the core of the first part of the seminar. The speaker should combine the previous result to carry out the construction of compact manifolds with holonomy G_2. It is more important to understand some simple examples properly (e.g. an example where the singular set of the orbifold is a submanifold, and not a stratified space) than to understand many examples. In Joyce's book this is [6, Section 11.3], but it might be wise to combine it with other literature, e.g. the overview articles `http://arxiv.org/abs/math/0203158`, [3] and [2], and with the original articles [1]. For the following part of the seminar, the alternative construction by Kovalev sketched in [6, Subsection 11.3.5] will become important. Again this can be done later.

Supplementary Talk no. 11: Constructions of compact manifolds with holonomy Spin(7) In this talk we explain a modification of the construction above which finally yields compact manifolds with holonomy Spin(7). This talk requires QALE spaces. Literature [6, Sections 11.4–11.6]. As above it is more important to understand some examples properly than to deal with all the examples listed in the book.

Seminar-Homepage

`http://www.mathematik.uni-regensburg.de/ammann/holonomy`

Literatur

[1] Joyce, D. Compact Riemannian 7-manifolds with holonomy G_2. I, II. J. Differential Geom. 43, 2 (1996), 291–328, 329–375.

[2] Joyce, D. Compact manifolds with exceptional holonomy. In Proceedings of the International Congress of Mathematicians, Vol. II (Berlin, 1998) (1998), no. Extra Vol. II, pp. 361–370.

[3] Joyce, D. Compact Riemannian manifolds with exceptional holonomy. In Surveys in differential geometry: essays on Einstein manifolds, Surv. Differ. Geom., VI. Int. Press, Boston, MA, 1999, pp. 39–65.

[4] Joyce, D. Asymptotically locally Euclidean metrics with holonomy $\mathrm{SU}(m)$. Ann. Global Anal. Geom. 19, 1 (2001), 55–73.

[5] Joyce, D. Quasi-ALE metrics with holonomy $\mathrm{SU}(m)$ and $\mathrm{Sp}(m)$. Ann. Global Anal. Geom. 19, 2 (2001), 103–132.

[6] JOYCE, D. Riemannian holonomy groups and calibrated geometry, vol. 12 of Oxford Graduate Texts in Mathematics. Oxford University Press, Oxford, 2007.

[7] JOYCE, D. D. Compact manifolds with special holonomy. Oxford Mathematical Monographs. Oxford University Press, Oxford, 2000.