

Seminar: Ricci flow

Winter term 2016/17

Prof. Bernd Ammann

Friday 8:30-10:00,

Dates: 21.10., 28.10., 4.11., 11.11., 18.11., 25.11., 2.12., 9.12., 16.12., 23.12., 13.1., 20.1., 27.1., 3.2., 10.2., (15 weeks)

1 Arbitrary Dimensions, Part I

Talk no. 1: Introduction. *21.10.*

Definition of the Ricci flow, mention short time existence and uniqueness (proof later), first results and typical effects, scalar version of the maximum principle ([1, 4.1, Pages 93–94] for the max. princ.).

2 Ricci flow for surfaces

Talk no. 2: Conformal changes and evolution of the curvature. *28.10.*
and *4.11.*

- a) Conformal changes of metrics on surfaces [1, 5.1, Pages 106–109]
- b) Evolution of the curvature [1, 5.2, Pages 109–111]

Talk no. 3: Ricci solitons for surfaces. *11.11.* and *18.11.*
[1, 5.3 and 5.4, Pages 111–119]

Talk no. 4: Convergence for nonpositive Euler number. *25.11.*

- a) Convergence for negative Euler number [1, 5.5, Pages 120–123]
- b) Convergence for vanishing Euler number [1, 5.6, Pages 123–128]

Talk no. 5: Convergence for positive Euler number, Overview and Surface entropy. *2.12.*

- a) Overview [1, 5.7, Pages 128–132].
- b) Surface entropy [1, 5.8, Pages 133–137]

Talk no. 6: Upper bounds for R and ∇R . *9.12.*
[1, 5.9, Pages 137–143]

Talk no. 7: Harnack estimates. *16.12.*
[1, 5.10, Pages 143–148]

Talk no. 8: Convergence for positive curvature and estimates for the injectivity radius. *23.12.*

Convergence for positive Gauß curvature [1, 5.11, Pages 148–149], behaviour of the injectivity radius [1, 5.12, Pages 149–153]

Talk no. 9: Convergence for sign changing curvature and positive Euler number. 13.1.

[1, 5.13, Pages 153–156]

3 Arbitrary dimensions, Part II

Details and fine tuning will come later

Talk no. 10: Short time existence and uniqueness in general.

Talk no. 11: Neck pinch. The original reference is [7]. The speaker should investigate whether he wants to follow [7] or the more recent reference [8]. Beware of the typos in the corresponding section of [1]. If time permits, it would be interesting to say some words about the degenerate neckpinch [9].

Talk no. 12: Curvature evolution.

Talk no. 13: Tensorial maximum principles. [1, Pages 95–104] Recall the scalar maximum principle. Then prove the tensorial generalizations.

Talk no. 14: Shi estimates.

4 Ricci flow in 3 dimensions

Talk no. 15: Geometrization of 3-manifolds.

Talk no. 16: Perelman’s entropy. Several talks

Seminar-Homepage

http://www.mathematik.uni-regensburg.de/ammann/lehre/2016w_ricci

Central Literature for dim=2

- [1] B. Chow, D. Knopf; The Ricci Flow: An Introduction, AMS 2004, E-Book link UB Regensburg
- [2] X. Chen, P. Lu, G. Tian; A note on uniformization of Riemann surfaces by Ricci flow, Proc. Amer. Math. Soc., **134**(2006) 3391–3393 (*electronic*), DOI 10.1090/S0002-9939-06-08360-2, <https://arxiv.org/abs/math/0505163>
- [3] R. Hamilton; The Ricci flow on surfaces, contained in “Collected papers on Ricci flow”, B. Ammann has a paper copy
- [4] J.A. Isenberg, R. Mazzeo, N. Sesum; Ricci flow in two dimensions, <https://arxiv.org/abs/1103.4669>

- [5] L. Ma; Ricci-Hamilton flow on surfaces, lecture notes, <http://faculty.math.tsinghua.edu.cn/~lma/lectures/ricciface2.pdf>
- [6] B. Stetler; The Ricci flow on surfaces and the uniformization theorem, Master-Arbeit

Central Literature for $\dim \geq 2$

- [7] M. Simon; A class of Riemannian manifolds that pinch when evolved by Ricci flow, *Manuscripta Math.* **101**, 89 – 114 (2000)
- [8] S. Angenent, D. Knopf; An example of neckpinching for Ricci flow on S^{n+1} . *Math. Res. Lett.* **11**, 493–518 (2004). DOI
- [9] S. Angenent, D. Knopf; Degenerate neckpinches in Ricci flow. *J. Reine Angew. Math.* **709**, 81–117 (2015).

Extended Literature

- [10] G. Besson; Preuve de la conjecture de Poincaré en déformant la métrique par la courbure de Ricci (d’après G. Perelman). *Séminaire Bourbaki*. Vol. 2004/2005. Astérisque No. 307 (2006), Exp. No. 947, ix, 309–347.
- [11] C. Böhm; B. Wilking; Manifolds with positive curvature operators are space forms. *Ann. of Math. (2)* **167** (2008), no. 3, 1079–1097.
- [12] M. Boileau, Geometrization of 3-manifolds with symmetries. Link siehe Homepage.
- [13] S. Brendle; Ricci Flow and the Sphere Theorem, *AMS Graduate Studies* Vol. 111, 2010
- [14] S. Brendle; R. Schoen; Manifolds with 1/4-pinched curvature are space forms. *J. Amer. Math. Soc.* **22** (2009), no. 1, 287–307.
- [15] S. Brendle; R. Schoen; Classification of manifolds with weakly 1/4-pinched curvatures, *Acta Math.* **200**, 1–13 (2008) See also ArXiv: 0705.3963
- [16] S. Brendle; R. Schoen; Curvature, sphere theorems, and the Ricci flow; to appear at *Bull. AMS*, ArXiv: 1001.2278
- [17] B. Chow; P. Lu; L. Ni; Hamilton’s Ricci flow; *Graduate Studies in Mathematics*, **77**; American Mathematical Society, Providence, RI; Science Press, New York, 2006
- [18] H.D. Cao, B. Chow, S.C. Chu, S.T. Yau (editeurs), *Collected Papers on Ricci Flow*, International Press, Series in Geometry and Topology, Volume 37

- [19] B. Chow, *J. Diff. Geom.*, **33**, (1991), pp. 325–334
- [20] B. Chow, S.-C. Chu, D. Glickenstein, C. Guenther, J. Isenberg, T. Ivey, D. Knopf, P. Lu, F. Luo, L. Ni, *The Ricci flow, techniques and applications, Part I Geometric Aspects*, AMS Math. surv. and Monographs, Vol. 135.
- [21] DeTurck, D. M.; *Deforming metrics in the direction of their Ricci tensors*; *J. Differential Geom.* 18 (1983), no. 1, 157–162.
- [22] A. Hatcher. *Notes on basic 3-manifold topology*, Link siehe Homepage.
- [23] B. Kleiner, J. Lott, *Notes on Perelman’s Paper*
- [24] J. Lott, *www-Seite*, <http://math.berkeley.edu/~lott/ricciflow/perelman.html>
- [25] J. Morgan, G. Tian, *Ricci Flow and the Poincaré Conjecture*, ArXiv: [math/0607607](https://arxiv.org/abs/math/0607607)
- [26] J. Morgan, *Recent progress on the Poincaré Conjecture and the classification of 3-manifolds*. *Bull. Amer. Math. Soc.* 42 (2005), 57-78. Link siehe Homepage
- [27] J. Milnor, *Towards the Poincaré Conjecture and the Classification of 3-Manifolds*, *Notices Amer. Math. Soc.* 50 (2003), no. 10, 1226–1233.
- [28] G. Perelman, *The entropy formula for the Ricci flow and its geometric applications*, ArXiv: [math.DG/0211159](https://arxiv.org/abs/math.DG/0211159)
- [29] G. Perelman, *Ricci flow with surgery on three-manifolds*, ArXiv: [math.DG/0303109](https://arxiv.org/abs/math.DG/0303109)
- [30] G. Perelman, *Finite extinction time for the solutions to the Ricci flow on certain three-manifolds*, ArXiv: [math.DG/0307245](https://arxiv.org/abs/math.DG/0307245)
- [31] T. Shioya, T. Yamaguchi, *Collapsing three-manifolds under a lower curvature bound*. *J. Differential Geom.* 56 (2000), no. 1, 1–66.
- [32] T. Shioya, T. Yamaguchi, *Volume collapsed three-manifolds with a lower curvature bound*, *Math. Ann.* 333 (2005), no. 1, 131–155.
- [33] P. Topping; *Lectures on the Ricci flow*, LMS Lecture Note series 325, Cambridge

Alternative literature list on [web site of the seminar](#).