Analysis and geometry of Hitchin's self-duality equations
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Abstract: The Theorem of Narasimhan and Seshadri states a correspondence between the moduli space of stable holomorphic vector bundles over a Riemann surface X and that of irreducible unitary connections of constant central curvature. This is one instance of a much more general correspondence due to Kobayashi and Hitchin. Higgs bundles come into play when the compact Lie group $\mathrm{SU}(r)$ is replaced by $\mathrm{SL}(r, \mathbb{C})$. A suitable generalization of the constant central curvature connections in the former case is found in the solutions to Hitchin's self-duality equations. Due to the noncompactness of the Higgs bundle moduli space, a whole set of new question arises:

- What is the degeneration profile of "large" solutions to the self-duality equations?
- How can it be compactified?
- How does its geometry look like "at infinity"?
- How does the moduli space change when the underlying surface X is varied?

I shall explain answers to some of these questions. The results of this talk are mainly based on joint work with Rafe Mazzeo, Hartmut Weiß and Frederik Witt.