Properties & applications of projective special real manifolds

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A projective special real (PSR) manifold $\mathcal{H} \subset \mathbb{R}^{n+1}$ is a hyperbolic centro-affine hypersurface that is contained in the level set of a hyperbolic homogeneous cubic polynomial. PSR manifolds are the scalar manifolds in 5d supersymmetry coupled to gravity, which relates them to projective special Kähler manifolds (via the supergravity r-map) and quaternionic Kähler manifolds (via the composition of the supergravity r- and c-map). They also appear in the study of the geometry of the cone of Kähler classes of compact Kähler 3-folds.

I will present a quick overview of the current state of research related to PSR manifolds and explain the construction of a generating set of the moduli set of complete connected PSR manifolds for every dimension $n \geq 1$ (two PSR manifolds are called equivalent if they are related by a linear transformation of the ambient space). Properties of that generating set can be used to find global curvature bounds for these manifolds, and one can show that the boundary behaviour of a PSR manifold is related to whether or not one representative is contained in the boundary of the aforementioned generating set. I will also briefly discuss possible generalizations of PSR manifolds related to higher degree homogeneous polynomials which naturally appear as hypersurfaces in the cone of Kähler classes of compact Kähler $n > 3$-folds, and explain some difficulties that one encounters in comparison with the PSR case. Lastly I will give an outlook in which I will discuss how to (possibly) relate a certain notion of "limit behaviour" of PSR manifolds to the study of the time-incomplete Kähler-Ricci flow.