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Universität Regensburg

Analysis on complex manifolds
Problem set 1

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Exercise 1: Let U be an open subset of $\mathbb{C}^n$ and let f, g be holomorphic functions on U. Show that $f + g$, $f \cdot g$ are holomorphic functions on U and that f/g is holomorphic on $\{g \neq 0\}$.

Exercise 2: Show that $\mathbb{P}^n_{\mathbb{R}}$ is a smooth manifold and that $\mathbb{P}^n_{\mathbb{C}}$ is a complex manifold.

Exercise 3: Generalize the above for the Grassmannian $G_{k,n}$ over $\mathbb{R}$ (resp. over $\mathbb{C}$).

Exercise 4: Show that a smooth closed algebraic subvariety of $\mathbb{P}^n_{\mathbb{C}}$ has a natural structure as a complex submanifold of $\mathbb{P}^n_{\mathbb{C}}$.

Exercise 5: Prove that every holomorphic function on a connected compact complex manifold is constant.