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Analysis on complex manifolds

Problem set 2

for 20.4.2016

Exercise 1: Show that a smooth (resp. holomorphic) vector bundle E over a smooth (resp. complex) manifold X has a unique structure as a smooth (resp. complex) manifold such that the local trivializations are diffeomorphisms (resp. bilholomorphic).

Exercise 2: Show that the tangent bundle T_M of a smooth n-dimensional manifold M is a smooth real vector bundle of rank n.

Exercise 3: Prove that the universal bundle over the Grassmannian $G_{r,n}(\mathbb{C})$ is a holomorphic vector bundle of rank r. For simplicity, you may restrict to the case $r = 1$.

Exercise 4: Let $f : E \to F$ be a homomorphism of smooth (resp. holomorphic) vector bundles over the smooth (resp. complex) manifold X. We suppose that the rank of the induced linear map $f_x : E_x \to F_x$ on the fibres over $x \in X$ is constant in x.

(a) Show that Ker(f) is a smooth (resp. holomorphic) subbundle of E;

(b) Show that $f(E)$ is a smooth (resp. holomorphic) subbundle of F.

Prove that the assumption on the rank is always satisfied for X connected.