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Analysis on complex manifolds

Problem set 4

for 4.5.2016

Exercise 1: Show that the sections of a vector bundle form a sheaf.

Exercise 2: Give an example of a sheaf on a complex manifold which is not of the form as in Problem 4.1.

Exercise 3: Let X be a complex manifold with sheaf of analytic functions $\mathcal{O}_X$. A sheaf $\mathcal{F}$ of $\mathcal{O}_X$-modules on X is called locally free of rank r if every $x \in X$ has a neighbourhood U such that the restriction of $\mathcal{F}$ to U is isomorphic to $\mathcal{O}_X^r$. Show that there is a natural bijective correspondence between locally free sheaves of $\mathcal{O}_X$-modules of rank $r < \infty$ and vector bundles on X of rank r.

Exercise 4: Let $\mathcal{F}$ be a presheaf on the topological space X and let $\mathcal{F}^+$ be the associated sheaf constructed in the course. We have also defined a homomorphism $\theta : \mathcal{F} \to \mathcal{F}^+$. Show that the pair $(\mathcal{F}^+, \theta)$ satisfies the following universal property: For any sheaf $\mathcal{G}$ and any homomorphism $\varphi : \mathcal{F} \to \mathcal{G}$, there is a unique homomorphism $\psi : \mathcal{F}^+ \to \mathcal{G}$ such that $\varphi = \psi \circ \theta$.