Prof. Dr. Walter Gubler
Sommersemester 2015/16
Universität Regensburg

Analysis on complex manifolds
Problem set 6

for 16.5.2016

Exercise 1: Show the isomorphism classes of holomorphic line bundles on a complex manifold X form a group which is isomorphic to $\check{H}^1(X, \mathcal{O}_X^\times)$, where $\mathcal{O}_X^\times$ is the sheaf of invertible analytic functions on X.

Exercise 2: Show Leray's theorem: Suppose that $\mathcal{U}$ is a covering of the (paracompact Hdf) topological space X and let $\mathcal{F}$ be a sheaf on X. We assume that for $k > 0$ and any finite intersection $V := U_{i_0} \cap \cdots \cap U_{i_p}$ of open subsets from $\mathcal{U}$, we have $H^k(V, \mathcal{F}) = 0$. Prove that the natural maps $\check{H}^p(\mathcal{U}, \mathcal{F}) \to H^p(X, \mathcal{F})$ are isomorphisms for all $p \geq 0$.

Exercise 3: Compute the Cech cohomology of the sheaf Ω^1 of holomorphic differentials on $\mathbb{P}^1_{\mathbb{C}}$. Hint: Hartshorne, Example 4.0.3 and Leray's theorem.

Exercise 4: Same job for the constant sheaf $\mathbb{Z}$ on the circle $S^1 = \{z \in \mathbb{C} \mid |z| = 1\}$.