Prof. Dr. Walter Gubler
Sommersemester 2015/16 Universität Regensburg

Analysis on complex manifolds
Problem set 7

for 23.5.2016

Exercise 1: Let E, F be smooth $\mathbb{C}$-vector bundles on the smooth manifold X with sheaf of sections $\mathcal{E}_E$ and $\mathcal{E}_F$. Show that there is a canonical isomorphism $\mathcal{E}_{E\otimes F} \cong \mathcal{E}_E \otimes_{\mathcal{E}_{\mathbb{C}}} \mathcal{E}_F$ of sheaves, where $\mathcal{E}_{\mathbb{C}}$ is the sheaf of complex valued C^∞-functions on X.

Exercise 2: A connection on the smooth $\mathbb{C}$-vector bundle E is a $\mathbb{C}$-linear function $D : \mathcal{E}(X) \to \mathcal{E}^1(X)$, where $\mathcal{E}$ is as above and more generallly $\mathcal{E}^p$ is the sheaf of smooth differential forms with coefficients in E. Show that D induces a canonical sheaf homomorphism $\mathcal{E} \to \mathcal{E}^1$ which agrees with D on X. Moreover, prove that this sheaf homomorphism induces a connection on every open subset U of X.

Exercise 3: Let E, F be as above. Show that any homomorphism $E \to F$ of vector bundles induces a sheaf homomorphism $\mathcal{E} \to \mathcal{F}$. Which sheaf homomorphisms arise in this way?

Exercise 4: Let D be a connection as above. Show that D is not arising from a homomorphism $E \to \Lambda^1 T_X^* \otimes E$ of vector bundles.