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Sommersemester 2015/16

Analysis on complex manifolds
Problem set 9

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Exercise 1: Let us consider the Gauss-distribution $f_0(x) = e^{-\frac{1}{2}\|x\|^2}$ on $\mathbb{R}^m$. Show that $\widehat{f_0} = f_0$ holds.

Exercise 2: Let f, g be Schwarz functions on $\mathbb{R}^m$. Prove that

$$\widehat{f} \cdot \widehat{g} = \widehat{f * g}, \quad \widehat{f} * \widehat{g} = \widehat{fg}$$

hold.

Exercise 3: Prove that $C_c^\infty(\mathbb{R}^m)$ is dense in the Sobolev space $H_s(\mathbb{R}^m)$ for every $s \in \mathbb{R}$.

Exercise 4: Let P be the pseudodifferential operator on $\mathbb{R}^m$ given by the symbol $p(x, \xi)$. We assume that the symbol has order d for every $d \in \mathbb{R}$ which means that $p \in S^{-\infty}(\mathbb{R}^m)$.

(a) Prove that P maps $H_s(\mathbb{R}^m)$ to $H_t(\mathbb{R}^m)$ for every $s, t \in \mathbb{R}$.

(b) Prove that P is a smoothing operator which means that $Pf \in C^\infty(\mathbb{R}^m)$ for every $f \in H_s(\mathbb{R}^m)$.