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Arakelov Geometry
Problem Set 1

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In the following, X is a complex manifold of dimension n.

Exercise 1 Show that the currents of type (p, q) form a sheaf $D^{p,q}$ on X.

Exercise 2 Show that $A_c^{n,n}(X) \to \mathbb{C},\ \alpha \mapsto \int_X \alpha$ defines a current $\delta_X \in D^{0,0}(X)$.

Exercise 3 For $\alpha \in A^{p,q}(X)$ and $T \in D_{r,s}(X)$, we define $\alpha \cap T : A_c^{r-p,s-q}(X) \to \mathbb{C}$ by $(\alpha \cap T)(\beta) := T(\alpha \cap \beta)$. Show that $\alpha \cap T \in D_{r-p,s-q}(X)$.

Exercise 4 A morphism $f : X \to Y$ of complex manifolds is called proper if the preimage $f^{-1}(C)$ of any compact subset C of Y is compact.

a) Prove that the analytification of a projective morphism of regular algebraic schemes over $\mathbb{C}$ is proper.

b) Use the Chow Lemma to generalize a) to proper morphisms of regular algebraic schemes where proper here is meant in the sense of algebraic geometry.

Exercise 5 Let $f : X \to Y$ be a proper morphism of complex manifolds. Show that pull-back of forms induces a continuous linear map $f^* : A_c^{p,q}(Y) \to A_c^{p,q}(X)$ of locally convex spaces and hence a dual map $f_* : D_{p,q}(X) \to D_{p,q}(Y)$.