Prof. Dr. Walter Gubler
Thomas Fenzl
Christian Vilsmeier
Veronika Wanner

Universität Regensburg
Sommersemester 2017

Arakelov Geometry

Problem Set 2

until May 9, 2017

Exercise 1 Show that the Cauchy kernel $\omega = \frac{dz}{2\pi i z} \in A^{1,0}(\mathbb{C}^*)$ is locally integrable on $\mathbb{C}$ with d-residue $\delta_{\{0\}} \in D^{1,1}(\mathbb{C})$.

Exercise 2 Let $z_1, \dots, z_n$ be the coordinates on $\mathbb{C}^n$. Show that $\log |z_1|^2$ is locally integrable on $\mathbb{C}^n$ and that the dd^c-residue of $\log |z_1|^2$ is $\delta_{\{z_1=0\}}$.
Hint: Use Stoke's theorem which also holds for ∂ and $\overline{\partial}$.

Exercise 3 Let $f : X \to Y$ be a morphism of projective complex varieties. Show for every m-cycle Z on X that $(f^{\mathrm{an}})_*(\delta_{Z^{\mathrm{an}}}) = \delta_{(f_*(Z))^{\mathrm{an}}}$ holds in $D_{m,m}(X(\mathbb{C}))$.

Exercise 4 Use the above problems and Hironaka's resolution of singularities to prove the Poincaré–Lelong formula for a meromorphic section of a hermitian line bundle.