Prof. Dr. Walter Gubler
Thomas Fenzl
Christian Vilsmeier
Veronika Wanner

Universität Regensburg
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Arakelov Geometry
Problem Set 3

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Exercise 1 Let $\overline{L} = (L, h)$ be a hermitian line bundle on the complex manifold X. Show that the definition of the first Chern form $c_1(L, h)$ in 1.2.16 does not depend on the choice of the local frame and gives a well-defined $(1, 1)$-form on X.

Exercise 2 In the setting of Exercise 1, let h' be another hermitian metric on L. Show that

$$c_1(L, h') - c_1(L, h) = d\eta$$

for $\eta \in A^1(X)$ and hence we get a well-defined element $c_1(L) := [c_1(L, h)] \in H^2_{\mathrm{dR}}(X)$ independent of the hermitian metric h. We call $c_1(L)$ the *first Chern form of* L.

Exercise 3 Let A be a 1-dimensional integral domain with field of fractions K. Prove for $f \in K^\times$ that

$$\mathrm{ord}_A(f) := l_A(A/aA) - l_A(A/bA)$$

is independent of the choice of $a, b \in A$ with $f = a/b$. Show that ord_A induces a homomorphism $K^\times \to \mathbb{Z}$.

Exercise 4 Let $\mathscr{X} := \mathrm{Spec}(\mathbb{Z}[x_1, x_2])$. Compute the relative dimension $\dim_S(\mathscr{X})$ of $\mathscr{X}$ over $S := \mathrm{Spec}(\mathbb{Z})$. Give examples of closed subvarieties $\mathscr{Y}$ of $\mathscr{X}$ of relative dimension $-1, 0, \ldots, \dim_S(\mathscr{X})$ and give also their topological dimensions.