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Arakelov Geometry

Problem Set 5

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Exercise 1 Give an example of a morphism $\varphi : X \to Y$ of smooth projective manifolds such that there is a differential form $\eta \in A^*(X)$ with $\varphi_*([\eta])$ not equal to the current associated to a smooth differential form on Y.

Exercise 2 Let X be a projective manifold. We consider a hermitian line bundle $(L, \| \ \|)$ on X with non-trivial meromorphic section s. Then $\eta_D := -\log \|s\|^{-2}$ is a logarithmic Green form for $D := \mathrm{div}(s)$. Let Z be any cycle of codimension p with Green current g_Z. We assume that D and Z intersect properly. Prove that $[\eta_D] * g_Z$ is a Green current for $D.Z$.

Exercise 3 Let $\mathfrak{X}$ be an arithmetic variety over the ring of integers $\mathcal{O}_K$ of a number field K and let $Z \in Z^p(\mathfrak{X})$. Use the results on Green currents on compact Kähler manifolds to show that there is a Green current g_Z for Z. The additional ingredients here are that g_Z has to be real and $(F_\infty)_*$-invariant.

Exercise 4 Let $\mathfrak{X}$ be an arithmetic variety as above and let (Z, g_Z) be an arithmetic cycle of codimension 1. Prove that there is a hermitian line bundle $(\mathfrak{L}, \| \ \|)$ with a non-trivial meromorphic section s such that $(Z, g_Z) = \widehat{\mathrm{div}}(s)$.
Moreover, if $(\mathfrak{L}', \| \ \|')$ is another hermitian line bundle with non-trivial meromorphic section s' such that $(Z, g_Z) = \widehat{\mathrm{div}}(s')$, then show that there is an isometry $(\mathfrak{L}, \| \ \|) \cong (\mathfrak{L}', \| \ \|')$ mapping s to s'.