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Arakelov Geometry

Problem Set 6

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Exercise 1 Let $\varphi\colon \mathfrak{X}' \to \mathfrak{X}$ be a morphism of arithmetic varieties over $\mathcal{O}_K$ for the number field K. We assume that the generic fibre $\varphi_\eta\colon \mathfrak{X}'_\eta \to \mathfrak{X}_\eta$ is a smooth morphism over K. Show that $\varphi_*(Z', g') := (\varphi_*(Z'), \varphi_*(g'))$ induces a well-defined map

$$\varphi_* : \widehat{\mathrm{CH}}(\mathfrak{X}') \to \widehat{\mathrm{CH}}(\mathfrak{X}).$$

Exercise 2 Let $\varphi\colon \mathfrak{X}' \to \mathfrak{X}$ be a flat morphism of arithmetic varieties over $\mathcal{O}_K$. Prove that $\varphi^*(Z, [\eta]) := (\varphi^* Z, [\varphi^*\eta])$ induces a well-defined map

$$\varphi^*\colon \widehat{\mathrm{CH}}^p(\mathfrak{X}) \to \widehat{\mathrm{CH}}^p(\mathfrak{X}')$$

where η is any logarithmic Green form for Z.

Exercise 3 Recall that on a Kähler manifold (X, ω), we have the Hodge identities

$$[L, d] = 0 \quad \text{and} \quad [L, d^*] = 4\pi d^c$$

(see [GH] or [Wells]), where $L\colon A^*(X) \to A^{*+2}(X); \eta \to \omega \wedge \eta$. Use that to prove that

$$[L, \Delta] = 0.$$

Here, [,] means the commutator.

Exercise 4 Prove that the Kähler form ω of a Kähler manifold is harmonic (hint: use Problem 3).