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Arakelov Geometry

Problem Set 7

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Exercise 1 Let X be a compact Riemann surface of genus $g \geq 1$ and let ω be a Kähler form on X with $\int_X \omega = 1$. Let $g(x, y)$ be the associated Green function. Use the axioms of g and Proposition 3.5.7 to show that g is symmetric.

Exercise 2 Let X be as above and let $P \in X$. Show that for any holomorphic line bundle L on X, there is a canonical isomorphism

$$\det\left(R\Gamma(X, L)\right) \cong \det\left(R\Gamma(X, L \otimes \mathcal{O}(-P))\right) \otimes \left(L\Big|_P\right).$$

Hint: Use the associated long exact cohomology sequence associated to the short exact sequence

$$0 \to L \otimes \mathcal{O}(-P) \overset{\otimes s_P}{\to} L \to L\Big|_P \to 0.$$

Exercise 3 (8 points) Let $\mathfrak{X}$ be an arithmetic surface over $\mathcal{O}_K$ for a number field K. Describe explicitly how we get a graded ring structure on the arithmetic Chow ring $\widehat{\mathrm{CH}}^*(\mathfrak{X})$. Use the construction of the arithmetic intersection product for arbitrary arithmetic varieties and give the simplifications based on $\dim(\mathfrak{X}) = 2$.