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Arakelov Geometry

Problem Set 9

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Exercise 1 Use the definition of ampleness and the induction formula to show that for ample hermitian line bundles $\overline{\mathfrak{L}}_0, \ldots, \overline{\mathfrak{L}}_d$ on an arithmetic variety $\mathfrak{X}$ and for a closed d-dimensional subvariety Y of $\mathfrak{X}$, we have

$$\widehat{\deg}(\hat{c}_1(\overline{\mathfrak{L}}_0) \cdots \hat{c}_1(\overline{\mathfrak{L}}_d) \cdot Y) > 0.$$

Exercise 2 Show that on any arithmetic variety, there is an ample hermitian line bundle.

Exercise 3 Let $\overline{\mathfrak{L}} := \overline{\mathscr{O}}_{\mathbb{P}^n_{\mathscr{O}_K}}(1)$ be endowed with the Fubini-metric. Prove that

$$h_{\overline{\mathfrak{L}}}(\mathbb{P}^n_{\mathscr{O}_K}) = \frac{1}{2} \cdot \sum_{j=1}^{n} \sum_{k=1}^{j} \frac{1}{k} \quad .$$

Hint: Use the induction formula and then [Stoll, Acta Math. 123, Lemma 2.1] to compute the integral.

Exercise 4 Let (X, φ, L) be a dynamical system over a number field K. For any $d \in \mathbb{N}$, work out the details for the existence of a canonical height $\hat{h}_L$ on $Z_d(X)$ given in Theorem 6.1.11.