The $K$-groups of a non-unital commutative ring $C$ are defined by
\[ K_i(C) = \pi_i \text{hofib}[K(C \times \mathbb{Z}) \to K(\mathbb{Z})] \quad (i \in \mathbb{Z}) \]
where on the right $K$ stands for the non-connective $K$-theory spectrum of a ring.

Let $A$ be a commutative ring and let $I \subset A$ be an ideal. In the early days of algebraic $K$-theory Bass constructed an exact sequence
\begin{equation}
K_1(A) \to K_1(A/I) \to K_0(I) \to K_0(A) \to K_0(A/I) \to K_{-1}(I) \to \cdots .
\end{equation}
Soon it was realized that in general one cannot expect this sequence to be continued on the left. This problem, known as excision in algebraic $K$-theory, was solved in [4] in characteristic zero and in [3] in general. In [1, Thm. 3.1] and in [2] a pro-version of excision is deduced.

**Theorem** (Suslin–Wodzicki, Suslin)
If $\text{Tor}_i^A(\mathbb{Z}, \mathbb{Z}) = 0$ for all $i > 0$ then (1) can be continued to the left, i.e. the natural map
\[ K_j(I) \xrightarrow{\sim} K_j(A, I) \]
is an isomorphism between $K$-theory of $I$ and relative $K$-theory of the pair $(A, I)$ for all $j \in \mathbb{Z}$.

**Corollary** (Geisser–Hesselholt, Morrow)
If $A$ is noetherian the map
\[ (K_j(I^n))_n \xrightarrow{\sim} (K_j(A, I^n))_n \]
is an isomorphism of pro-groups in $n$.

**References**


E-mail address: moritz.kerz@mathematik.uni-regensburg.de