Oberseminar "Systolic and filling inequalities in Riemannian geometry"

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In a large class of Riemannian manifolds, there is a remarkable connection between so-called systoles (i.e., the length of shortest non-contractible loops) and the volume: Gromov's systolic inequality. In this seminar, we will discuss proofs of the systolic inequality, as well as applications and relations to other fields.

Some familiarity with basic Riemannian geometry and algebraic topology is required to follow this seminar.

Talks

Talk 1 (Introduction)**.** We will watch and discuss Guth's talk about the systolic inequality at the Institute for Advanced Study, Princeton. We recommend that all participants read Berger's short introduction into systolic geometry before the first meeting.
Literature: [7], [2], [8]

Talk 2 (Systolic inequality for tori)**.** Precise definition of systole; examples and basic properties; proof of the systolic inequality for tori.
Literature: [6]

Talk 3 (Aspherical/essential manifolds)**.** Definition of aspherical spaces; examples of aspherical manifolds; definition of essential manifolds; examples of essential manifolds; relation between essentialness and enlargeability.
Literature: [10], [5, Appendix]

Talk 4 (Filling radius and strategy of proof of the general systolic inequality)**.** General systolic inequality; strategy of proof via filling radius; precise definition of filling radius; proof of upper the estimate of the systole in terms of the filling radius.
Literature: [4, Sections 0 and 1], [5, Section 2], [11]

Talk 5 (Estimating the filling volume, Wenger's proof)**.** (Filling) volume of Lipschitz chains; Wenger's proof of the lower estimate of the volume of Lipschitz cycles in terms of the filling volume ("isoperimetric inequality"); relation to the general systolic inequality.
Literature: [14], [4]

Talk 6 (Estimating the filling radius, Guth's generalisation; part 1)**.** This and the next talk should cover the following topics: Guth's estimates of balls in large Riemannian manifolds (including proofs); in particular, the technique of looking at the nerves of certain nice coverings should be explained in detail; relation of these results to the general systolic inequality; relation to entropy, minimal volume and simplicial volume.
Literature: [9], [13], [3], [12]

Talk 7 (Estimating the filling radius, Guth's generalisation; part 2)**.** [see above]

Talk 8 (Systolic freedom)**.** Higher dimensional "systoles"; systolic freedom. *Literature:* [1, Chapter 7.2.3]

Talk 9 (Related topics in geometric group theory)**.** The precise topics and references will be given later in the semester.

Talk 10 (Applications)**.** The precise topics and references will be given later in the semester.

Literatur

[1] M. Berger. *A panoramic view of Riemannian geometry*, Springer, 2003.

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[6] L. Guth. Systolic inequalities and minimal hypersurfaces, *Geom. Funct. Anal.*, 19(6), pp. 1688–1692, 2010.

[7] L. Guth. *Metaphors in Systolic Geometry*, talk at the Institute for Advanced Study, Princeton, 2010.
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[8] L. Guth. Metaphors in systolic geometry, preprint, available online at arXiv:1003.4247v1 [math.DG], 2010.

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[10] B. Hanke, D. Kotschick, J. Roe, T. Schick. Coarse topology, enlargeability, and essentialness, *Ann. Sci. Éc. Norm. Supér. (4)*, Vol. 41 (3), pp. 471–493, 2008.

[11] M.G. Katz. *Systolic geometry and topology*, With an appendix by J.P. Solomon. *Mathematical Surveys and Monographs*, 137, American Mathematical Society, 2007.

[12] C. Löh. Simplicial volume, *Bull. Man. Atl.*, pp. 7–18, 2011.
http://www.manifoldatlas.him.uni-bonn.de/Simplicial_volume

[13] R. Sauer. Amenable covers, volume and L^2-Betti numbers of aspherical manifolds, *J. Reine Angew. Math.*, 636, pp. 47–92, 2009.

[14] S. Wenger. A short proof of Gromov's filling inequality, *Proc. Amer. Math. Soc.*, 136(8), pp. 2937–2941, 2008.