Group Cohomology – Etudes

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Exercise 1 (universal coefficients). Compute the following homology groups (where the groups act trivially on the coefficients):

1. $H_*(S_3; \mathbb{F}_2)$
2. $H_*(S_3; \mathbb{F}_3)$
3. $H_*(D_\infty; \mathbb{F}_2)$
4. $H_*(D_\infty; \mathbb{F}_3)$

Exercise 2 (product groups). Compute the following homology groups (where the groups act trivially on the coefficients):

1. $H_*(\mathbb{Z}/2019 \times \mathbb{Z}/2019; \mathbb{F}_3)$
2. $H_*(\mathbb{Z}/3 \times \mathbb{Z}/2019; \mathbb{F}_3)$
3. $H_*(\mathbb{Z}/3 \times \mathbb{Z}/9; \mathbb{F}_3)$
4. $H_*(\mathbb{Z}/2019 \times \mathbb{Z}/2019; \mathbb{F}_{2017})$ (be lazy!)

Exercise 3 (algebraic topology). Recall the following terminology/facts:

1. contractibility
2. covering map
3. classification of coverings
4. CW-complex
5. singular homology (and its properties)
6. cellular homology (and its properties)

Hints. In case you don't know anything about algebraic topology: Don't panic! I will quickly review some basics in the lectures. However, it might still be helpful to browse literature on algebraic topology to get a first impression.

Exercise 4 (summary). Write a summary of Chapter 3.1 (The Hochschild-Serre spectral sequence), keeping the following questions in mind:

1. What are spectral sequences? How do they work?
2. Which spectral sequences are in your toolbox?
3. Which computational tricks do you know for spectral sequences?
4. How can spectral sequences be used in group (co)homology?
5. Did you check all the little things that we did not discuss in details in the lectures?

no submission!