

Group Cohomology – Etudes

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Exercise 1 (group presentations and amenability). Which of the following groups are amenable?

1. $\langle a, b \mid a, b \rangle$
2. $\langle a, b \mid aba^{-1}b^{-1} \rangle$
3. $\langle a, b, c \mid c^2 \rangle$
4. $\langle a, b, c \mid ab \rangle$
5. $\langle a, b \mid a^2, b^2 \rangle$
6. $\langle a \mid a^{2019} \rangle$

Exercise 2 (bounded cochains). Which of the following cochains in $\overline{C}^2(\mathbb{Z}^2; \mathbb{R})$ are bounded? Which of them are cocycles?

1. $[x \mid y] \mapsto \det(x \ y)$
2. $[x \mid y] \mapsto \|x\|_2$
3. $[x \mid y] \mapsto 2019$
4. $[x \mid y] \mapsto \frac{1}{\|x\|_2^{2020} + \|y\|_2^{2020} + 1}$

Exercise 3 (amenable vs. free). Start a table for amenable groups and free groups, respectively, listing their behaviour with respect to:

1. uniformly finite homology
2. bounded cohomology
3. quasi-morphisms
4. stable commutator length

Exercise 4 (summary). Write a summary of Chapter 2.2 (Uniformly finite homology) keeping the following questions in mind:

1. What is uniformly finite homology of spaces?
2. Which coefficients for group homology lead to uniformly finite homology?
3. What is the fundamental class in uniformly finite homology in degree 0?
4. How does all this relate to amenability?
5. Did you check all the little things that we did not discuss in detail in the lectures?

no submission!