Group Cohomology – Etudes

Prof. Dr. C. Löh/D. Fauser/J. P. Quintanilha/J. Witzig Sheet 9, June 27, 2019

Exercise 1 (scl)**.** Determine whether the following stable commutator lengths are zero or not:

1. $\operatorname{scl}_{\mathbb{Z}}\bigl([2019, 2018] \cdot [2019, 2018]\bigr)$
2. $\operatorname{scl}_{\langle a,b \,|\, \rangle}\bigl([a, b] \cdot [a, b]\bigr)$
3. $\operatorname{scl}_{\langle a,b \,|\, \rangle}\bigl([a \cdot b, b]\bigr)$
4. $\operatorname{scl}_{\langle a,b \,|\, \rangle}\bigl([a \cdot a, a]\bigr)$
5. $\operatorname{scl}_{\langle a,b \,|\, a^2, b^2\rangle}\bigl([a, b] \cdot [a, b]\bigr)$

Exercise 2 (Tor)**.** Compute the following Tor-groups!

1. $\operatorname{Tor}_0^{\mathbb{Z}}(\mathbb{Z}/2019, \mathbb{Z})$
2. $\operatorname{Tor}_{2019}^{\mathbb{Z}}(\mathbb{Z}/2019, \mathbb{Z}/2019)$
3. $\operatorname{Tor}_1^{\mathbb{Z}}(\mathbb{Z}, \mathbb{Z}/2019)$
4. $\operatorname{Tor}_1^{\mathbb{Z}}(\mathbb{Z}/2018, \mathbb{Z}/2019)$
5. $\operatorname{Tor}_1^{\mathbb{Z}}(\mathbb{Z}/2019, \mathbb{Z}/2019)$
6. $\operatorname{Tor}_1^{\mathbb{Z}}(\mathbb{Z}/2019, \mathbb{Q})$

Exercise 3 (quasi-isomorphisms)**.** For each choice C_* and D_* of the following chain complexes (over the ring $\mathbb{Z}$), decide whether there exists a quasi-isomorphism $C_* \longrightarrow D_*$ or not.

1. $\cdots \longrightarrow 0 \longrightarrow \mathbb{Z} \xrightarrow{\;0\;} \mathbb{Z} \longrightarrow 0$
2. $\cdots \longrightarrow 0 \longrightarrow \mathbb{Z} \xrightarrow{\;1\;} \mathbb{Z} \longrightarrow 0$
3. $\cdots \longrightarrow 0 \longrightarrow 0 \longrightarrow \mathbb{Z} \longrightarrow 0$
4. $\cdots \longrightarrow 0 \longrightarrow 0 \longrightarrow \mathbb{Z}/2019 \longrightarrow 0$
5. $\cdots \longrightarrow 0 \longrightarrow \mathbb{Z} \xrightarrow{\;2019\;} \mathbb{Z} \longrightarrow 0$

Exercise 4 (homological algebra)**.** Recall/look up the following fundamental results of homological algebra:

1. long exact homology sequence/snake lemma
2. horseshoe lemma

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no submission!