Exercise 1 (separation theorems?). Prove or disprove:

1. If \( f : S^1 \to \mathbb{R}^{2016} \) is continuous and injective, then \( \mathbb{R}^{2016} \setminus f(S^1) \) is path-connected.

2. If \( f : S^1 \to S^1 \times S^1 \) is continuous and injective, then \( S^1 \times S^1 \setminus f(S^1) \) has exactly two path-connected components.

Exercise 2 (connectivity).

1. Let \( n \in \mathbb{N} \). Show that an open subset of \( \mathbb{R}^n \) is connected if and only if it is path-connected.

2. Let \( X \) be a topological space that has only finitely many connected components. Show that the connected components of \( X \) are all open.

Exercise 3 (vanishing \( \ell^1 \)-semi-norm). In the following, we consider the \( \ell^1 \)-semi-norm \( \| \cdot \|_1 \) on \( H_\ast(C; \mathbb{R}) \) as defined in Exercise 3 of Sheet 11.

1. Let \( n \in \mathbb{N} \) and let \( k \in \mathbb{N}_{>0} \). Show that \( \| \cdot \|_1 \) is the zero semi-norm on \( H_k(S^n; \mathbb{R}) \).

2. Let \( k \in \mathbb{N}_{>0} \). Show that \( \| \cdot \|_1 \) is the zero semi-norm on \( H_k(S^1 \times S^1; \mathbb{R}) \).

Exercise 4 (homotopy groups of spheres). Let \( n \in \mathbb{N}_{>0} \). Calculate \( \pi_k(S^n, e^n_1) \) and \( \pi_k((S^n, e^n_1) \vee (S^n, e^n_1)) \) for all \( k \in \{2, \ldots, n\} \) via the Hurewicz theorem.

Bonus Problem (Hurewicz theorem). Fill in the details of the proof of the Hurewicz theorem. Illustrate your arguments with appropriate pictures!